

$$\begin{aligned}
 & 4^3 + 5^3 + 6^3 + 7^3 + 8^3 \\
 & \left(\sum_{n=4}^8 n^3 \right) \quad \begin{array}{c} 2+4+6+8+10+12 \\ n: 1 \ 2 \ 3 \ 4 \ 5 \ 6 \end{array} \\
 & \quad \quad \quad \sum_{n=1}^6 2n \\
 & \quad \quad \quad 2+4+6+8+10+12 \\
 & \quad \quad \quad n: 4 \ 5 \ 6 \ 7 \ 8 \ 9 \\
 & \quad \quad \quad \sum_{n=4}^9 2(n-3)
 \end{aligned}$$

$$1 - 3 + 5 - 7 + 9 - 11$$

$$\begin{aligned}
 & 1 + 3 + 5 + 7 + 9 + 11 \\
 & \begin{array}{c} n: 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\ 2n: 2 \ 4 \ 6 \ 8 \ 10 \ 12 \end{array} \\
 & \left(\sum_{n=1}^6 2(n-1) \right) - 1 \\
 & \quad \quad \quad \sum_{n=1}^6 (-1)^{n+1} (2n-1)
 \end{aligned}$$

$$\sum_{k=1}^n k =$$

if $n=1$

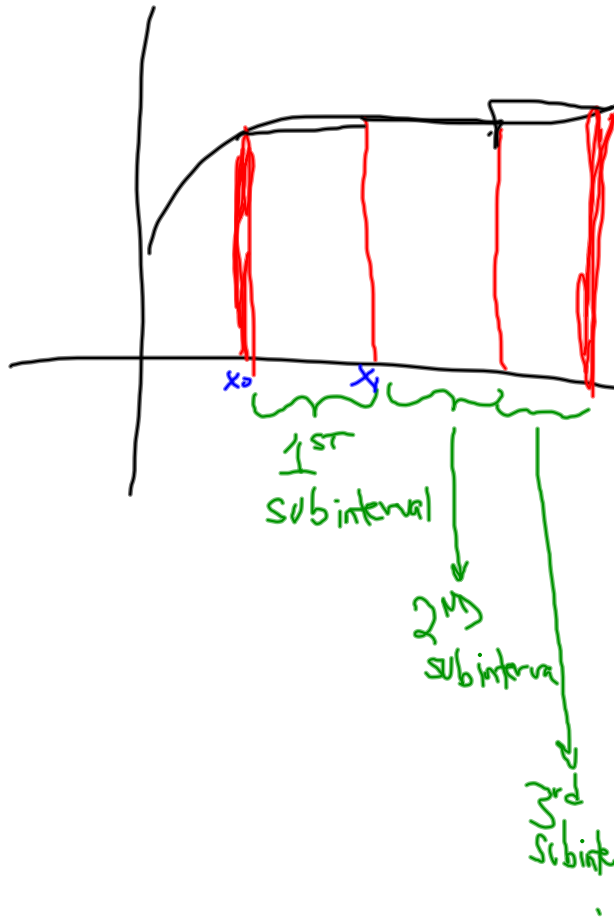
$$\sum_{k=1}^1 k = 1$$

if $n=3$

$$\sum_{k=1}^3 k = 1+2+3$$

if $n=2$

$$\sum_{k=1}^2 k = 1+2$$



$$\text{Rectangle 1: } \Delta x_1 \cdot f(x_1^*)$$

$$\text{Rectangle 2: } \Delta x_2 \cdot f(x_2^*)$$

$$\vdots$$
$$\text{Rectangle } i: \Delta x_i \cdot f(x_i^*)$$
$$\text{--- } w \text{ --- } h \text{ ---}$$
$$\vdots$$

Specific
Approximations
you will need
to know

Left endpt approx

$$(x_i^*) = (\text{left end pt of that subinterval})$$

Right endpt approx

$$x_i^* = \text{right end pt of that subinterval}$$

Mid point approx

$$x_i^* = \text{mid pt of the } x\text{-subinterval}$$

Exact
Area

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \Delta x_k f(x_k^*)$$

$$4^3 + 5^3 + 6^3 + 7^3 + 8^3$$

$$\sum_{k=4}^8 k^3$$

$$k: \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$2+4+6+8+10+12$$

$$\sum_{k=1}^6 2(k)$$

$$1+3+5+7+9+11$$

$$k: \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$2k: \begin{array}{cccccc} 2 & 4 & 6 & 8 & 10 & 12 \end{array}$$

$$\sum_{k=1}^6 2k-1$$

$$2+4+6+8+10+12$$

$$k: \begin{array}{cccccc} 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$

$$\sum_{k=4}^9 2(k-6)$$

$$8 \quad 10 \quad 12 \quad 14 \quad 16 \quad 18$$

$$1-3+5-7+9-11$$

$$k: \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$\sum_{k=1}^6 (-1)^{(k-1)} (2k-1)$$

$$\sum_{K=1}^n K$$

if $n=4$

$$\sum_{K=1}^4 K = 1+2+3+4$$

⋮

if $n=1$

$$\sum_{K=1}^1 K = 1$$

if $n=2$

$$\sum_{K=1}^2 K = 1+2$$

if $n=3$

$$\sum_{K=1}^3 K = 1+2+3$$

$$\sum_{k=1}^n k = 1 + 2 + 3 + \dots + (n-1) + n$$

$\sum_{k=1}^n k =$

	1	2	3	4	5	6	
	1	3	6	10	15	21	...
		2	3	4	5	6	...
diff=1							
diff=2							
k^2	1	4	9	16	25	36	
$\sum -k^2$	0	-1	-3	-6	-10	-15	...

$$\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + \dots + n^2$$

	1	5	14	30	55	91
D1		4	9	16	25	36
D2			5	7	9	11
D3				2	2	

rule = $\frac{2}{6}x^3 + \dots$

	1	5	14	30	55	91
$\frac{1}{3}x^3$	$\frac{1}{3}$	$\frac{8}{3}$	9	$\frac{64}{3}$	$\frac{125}{3}$	72
black-red	$\frac{2}{3}$	$\frac{7}{3}$	$\frac{15}{3}$	$\frac{26}{3}$	$\frac{40}{3}$	$\frac{57}{3}$
D1		$\frac{8}{3}$	$\frac{11}{3}$	$\frac{14}{3}$	$\frac{17}{3}$	
D2			1	1	1	
D3						

(1) $\frac{1}{2}x^2 + \dots$

	$\frac{2}{3}$	$\frac{8}{3}$	$\frac{15}{3}$	$\frac{26}{3}$	$\frac{40}{3}$	$\frac{57}{3}$
$\frac{1}{2}x^2$	$\frac{1}{2}$	2	$\frac{9}{2}$	8	$\frac{25}{2}$	18
subtract:	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$
D1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

original rule: $\frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x = \frac{2x^3 + 3x^2 + x}{6}$

$$\frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + 3n^2 + n}{6}$$

6.4 examples

2010-10-27 Pd 3



6.4 examples

2010-10-27 Pd 3

